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AUTHOR Dickinson, Terry L.
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ABSTRACT

The general linear model was described, and the influence that measurement errors have on model parameters was discussed. In particular, the assumptions of classical true-score theory were used to develop algebraic relationships between the squared multiple correlations coefficient and the regression coefficients in the infallible and fallible models. In addition, a Monte Carlo study was conducted to describe the changes in the infallible parameters due to measurement errors in the criterion variable and the predictor variables. The results indicated that the fallible squared multiple correlation was always less than its infallible counterpart, while the fallible regression coefficients either were greater than or less than their counterparts, depending on the size of their reliability of the criterion relative to the reliabilities of the predictors. The results were discussed within the context of regression analysis, analysis of variance, and analysis of covariance. (Author/GDC)

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The General Linear Model and Errors of Measurement

Terry L. Dickinson

Old Dominion University

Paper Presented at the Annual Meeting of the

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ABSTRACT

This paper describes the general linear model (GLM) and the influence that measurement errors have on model parameters. In particular, the assumptions of classical true-score theory were used to develop algebraic relationships between the squared multiple correlation coefficient and the regression coefficients in the infallible and fallible models. In addition, a monte carlo study was conducted to describe the changes in the infallible parameters due to measurement errors in the criterion variable and the predictor variables. The results indicated that the fallible squared multiple correlation was always less than its infallible counterpart, while the fallible regression coefficients either were greater than or less than their counterparts, depending on the size of the reliability of the criterion relative to the reliabilities of the predictors. The results were discussed within the context of regression analysis, analysis of variance, and analysis of covariance.

Key words: Measurement errors, general linear model, squared multiple correlation, and regression coefficients.

The General Linear Model and Measurement Errors

How do measurement errors affect the statistical procedures which are commmonly used in the social sciences? Since social scientists typically use fallible measures this is an important question to address. This paper considers the question in the framework of the general linear model (GLM). The GLM includes as special cases the procedures of regression analysis, analysis of variance, and analysis of covariance.

For infallible variables, the GLM is defined as:

$$\tau = T\phi + \Delta. \quad (1)$$

In this model, τ is a n x 1 matrix of true scores of a criterion variable, T is a n x k matrix of true scores of predictor variables, ϕ is a k x 1 parametric matrix of regression coefficients, and Δ is a n x 1 matrix of deviations from the model. The deviations from the model are distributed such that they are independent of each other and have a common mean of zero and a common variance.

Now, the GLM for variables containing measurement errors is defined as:

$$Y = X\beta + D. \quad (2)$$

In this fallible model, Y is a n x 1 matrix of observable scores of the criterion variable, X is a n x k

matrix of observable scores for k predictor variables, β is a k x 1 parametric matrix of regression coefficients, and D is a n x 1 matrix of deviations from the model. The deviations from the model are distributed such that they are independent of each other and have a common mean of zero and a common variance.

Independence Assumptions for Measurement Errors

Now, the assumptions of classical true-score theory are employed to relate the infallible and fallible models (cf. Allen & Yen, 1979; Lord & Novick, 1969). The criterion variables defined in equations (1) and (2) are related as:

$$Y = \tau + \varepsilon. \quad (3)$$

That is, an observed score is equal to true score plus error of measurement. The matrices in equation (3) have dimensions n x 1. The matrix ε contains measurement errors for n people on the criterion variable. It is assumed that the errors are distributed independently of one another and that they have a common mean of zero and a common variance. In addition, the errors are assumed to be independent of true scores. For simplicity of presentation, the observed scores are required to have a mean of zero and a variance of one.

The predictor variable of equations (1) and (2) are

related as:

(4) $$X = T + E.$$

This formulation also follows from the assumptions of classical true-score theory that an observed score is equal to true score plus error of measurement. The matrices in equation (4) are of dimensions n x k. The matrix E contains the measurement errors of n people for each of k predictor variables. Again, measurement errors are assumed to be distributed independently of one another and of the true scores. The independence of errors includes errors by the same person on different variables as well as errors by different people on the same or different variables. In addition, the errors for a predictor variable are assumed to have a common mean of zero and a common variance. Obviously, the predictor variables may have different measurement error variances. Furthermore, the observed scores of a predictor variable are required to have a mean of zero and a variance of one.

A final set of assumptions concerns the relationships between the true scores and measurement errors of the predictor and criterion variables. Namely, errors in the predictor variables are assumed to be indpendent of the errors in the criterion variable. In addition, measurement errors in predictor and criterion variables

are assumed to be independent of all true scores.

The assumptions that classical true-score theory makes about measurement errors can be used to derive a definition of reliability of measurment (cf. Allen & Yen, 1979; Lord & Novick, 1969). For the present purposes, the results of these derivations are stated. Namely,

$$\rho_{X_j X_j} = \rho^2_{X_j T_j} \quad \text{for } j = 1,2,\ldots,k \tag{5}$$

and

$$\rho_{YY} = \rho^2_{Y\tau}. \tag{6}$$

That is, the reliability of any fallible variable is defined as its squared correlation with the corresponding infallible variable. Furthermore, the correlations between the criterion and predictor variables in the fallible model are defined in terms of these reliabilities. Namely,

$$\rho_{X_j Y} = \rho_{T_j \tau}\, \rho_{X_j T_j}\, \rho_{Y\tau} \quad \text{for } j = 1,2,\ldots,k \tag{7}$$

Equation (7) is often referred to as providing a correction for attentuation due to errors of measurment.

Measurement Errors and The Fallible Model

The fallible model is now described in terms of the infallible model. The impact of measurement errors is

described only for population parameters. The predictor correlation matrix of the fallible model (R_{XX}) can be defined in terms of an "adjusted" true-score predictor correlation matrix (R^*_{TT}) defined for the infallible model. Namely,

$$R_{XX} = R_{XT} R^*_{TT} R_{XT} . \quad (8)$$

In equation (8), R_{XT} is a k x k diagonal matrix with the square roots of predictor reliabilities in its diagonal. Furthermore, R^*_{TT} is a k x k matrix with the correlations between true-score predictor variables in the off-diagonals and the reciprocals of the predictor reliabilities in the diagonal.

Next, the correlation matrix between fallible criterion and predictor variables (R_{XY}) is defined in terms of the predictor variables ($R_{T\tau}$). Here, we rely on equation (7). Namely,

$$R_{XY} = R_{XT} R_{T\tau} \rho_{Y\tau} . \quad (9)$$

In equation (9), $\rho_{Y\tau}$ is the square root of the reliability of the criterion variable.

Now, the squared multiple correlation coefficient and the regression coefficients are described for the infallible and fallible models. From the theory of least squares, the regression coefficients for the

infallible model are defined as

$$\phi = R_{TT}^{-1} R_{T\tau}, \tag{10}$$

and the coefficients for the fallible model are defined as

$$\beta = R_{XX}^{-1} R_{XY}. \tag{11}$$

In equation (11), R_{XX}^{-1} is the inverse of the predictor correlation matrix, and R_{XY} is the correlation matrix between the criterion and predictors. Now, we substitute into equation (11) the equalities defined in equations (8) and (9). Upon simplification we obtain:

$$\beta = R_{XT}^{-1} R^{*-1}_{TT} R_{T\tau} \rho_{Y\tau}. \tag{12}$$

The squared multiple correlation coefficient is defined from the theory of least squares as:

$$\rho^2_{Y.X_1,\ldots,X_2} = \beta' R_{XY}. \tag{13}$$

In equation (13), the results of equation (12) are substituted to obtain:

$$\rho^2_{Y.X_1,\ldots,X_k} = R_{T\tau}' R^{*-1}_{TT} R_{T\tau} \rho^2_{T\tau}. \tag{14}$$

Of course, the squared multiple correlation of the infallible model is defined as:

$$(15) \qquad \rho^{2}_{\tau.T_1,\ldots,T_k} = R_{T\tau}' R_{TT}^{-1} R_{T\tau}.$$

A comparison of equations (14) and (15) indicates the impact of measurement errors on the squared multiple correlation coefficient in the fallible model. The two equations differ in several respects. First, the inverse in equation (14) is R^{*-1}_{TT} and that of equation (15) is R^{-1}_{TT}. The matrices differ only by their diagonal entries. The diagonal of R_{TT} contains ones, while the diagonal of R^{*}_{TT} contains the reciprocals of the square roots of predictor reliabilities. Thus, the diagonal entries of R^{*}_{TT} are greater than the diagonal entries of R_{TT}, whenever the predictors contain measurement errors. Both R^{-1}_{TT} and R^{*-1}_{TT} are used with $R_{T\tau}$ to define weighted sums. Which weighted sum is smaller? The weighted sum taken on R^{*-1}_{TT} should be smaller. This occurs because the larger the diagonal entries in a matrix (with the remaining entries staying constant), the smaller will be <u>all</u> the entries in the inverse of that matrix. Thus, the size of $R_{T\tau}' R^{*-1}_{TT} R_{T\tau}$ is always less than $R_{T\tau}' R^{-1}_{TT} R_{T\tau}$.

A second way in which equations (14) and (15) differ is that equation (14) contains the multiplier $\rho^{2}_{Y\tau}$.

Criterion reliability is always between zero and one, and it further serves to reduce the size of $R_{T\tau}' R^{*-1}_{TT} R_{T\tau}$.

The two differences between equations (14) and (15) imply that the squared multiple correlation coefficient for the fallible model will be smaller than the same coefficient for the infallible model.

Finally, a comparison of equations (10) and (12) indicates that the entries in the matrix product $R^{*-1}_{TT} R_{T\tau}$ will be smaller than those in the product $R^{-1}_{TT} R_{T\tau}$. Furthermore, the product $R^{-1}_{XT} \rho_{Y\tau}$ is a diagonal matrix of the ratios of the square roots of the criterion's reliability to the respective predictor reliabilities. These ratios will be greater than or less than one, depending on whether the criterion is more reliable than the particular predictor. In sum, the fallible regression coefficients can be smaller or larger than the infallible regression coefficients, depending on the size of the reliability of the criterion relative to the reliabilities of the predictors.

Monte Carlo Study

In order to better understand the relationships between the infallible and fallible models, a monte carlo study was done to describe the variations in the squared multiple corelation coefficient and regression

coefficients for a GLM with two predictor variables. The study design is displayed in Table 1. As can be seen nine data sets were defined for the infallible model by variations in the squared multiple correlation coefficient (.30, .50, .80) and the squared correlations between the two predictors (.00, .16, .64). Variations in the reliability of the criterion (.30, .50, .80, 1.00) and the reliabilities of the two predictors (.30, .50, .80, 1.00) were introduced to define data sets for the fallible model.

Insert Table 1 about here

The Impact of Measurement Errors on the Squared Multiple Correlation Coefficient

Table 2 displays the squared multiple correlation coefficients that were obtained for the infallible data sets. In this table, correlation coefficients are not reported for the complete crossing of the reliabilities of the two predictors to remove redundancies. For example, the nine correlations for X_1 with reliability of .30 and X_2 with .50 are identical to those for X_1 with reliability of .50 and X_2 with .30.

Insert Table 2 about here

Inspection of the table suggests that when predictors are uncorrelated (data sets 1, 2, and 3), measurement errors in the predictors reduce the squared multiple correlation coefficient by a multipler consisting of the arithmetic average of predictor reliabilities. For example, compare the three data sets for Y with reliability .50 and the Xs with reliability 1.00 to other data sets with Y of that reliability. This conclusion emphasizes the point that uncorrelated predictors are desirable in that attenuation in the fallible squared multiple correlation can readily be corrected by knowledge of predictor reliabilities.

When predictors are correlated (data sets 4 through 9), measurement errors in the predictors reduce the squared multiple correlation coefficient by a multiplier consisting of a weighted average of reliabilities. The exact nature of the weighted average is specified by the "adjusted" true-score predictor correlation matrix. Furthermore, the reduction is less than when predictors are uncorrelated. For example, compare the nine data sets (1 vs 4 vs 7; 2 vs 5 vs 8; and 3 vs 6 vs 9) for any instance of measurement errors in the predictors. The

magnitude of the squared multiple correlation coefficient increases as the predictors become more intercorrelated.

The table also illustrates the impact of measurement errors in the criterion on the squared multiple correlation coefficient. The squared correlation is reduced by a multipler consisting of the criterion's reliability. This is best illustrated by comparing the data sets in which Y has perfect reliability to respective data sets in which Y has measurement errors. For example, the data set of Y with reliability of 1.00, X_1 of .50, and X_2 of .80 is reduced by the multipler of .50 to yield the data set of Y with reliability of .50, X_1 of .50, and X_2 of .80. Furthermore, data sets with perfectly reliable predictors and an unreliable criterion illustrate the influence of measurement errors in the analysis of variance. Measurement errors reduce the variation explained in Y with the treatment effects by the multiplier of criterion reliability.

Finally, the table illustrates that measurement errors in the predictors tend to reduce the squared multiple correlation coefficient less than do comparable errors in the criterion. This is due to the

intercorrelation between the predictors such that the greater the intercorrelation, the closer in magnitude are the fallible and infallible squared multiple correlation coefficients. For example, compare the data sets in which Y has reliability consisting of 1.00 and the Xs of .50 to the data sets in which Y has a reliability consisting of .50 and the Xs of 1.00.

The Impact of Measurement Errors on the Regression Coefficients

Table 3 displays the regression coefficients that were obtained for the infallible data sets. Inspection of the table suggests that when the reliabilities of the predictors are equal, the fallible regression coefficients are always reduced in magnitude to the same degree. That is, both coefficients are equally and consistently less than their true-score counterparts.

Insert Table 3 about here

When the predictors are uncorrelated, the regression coefficient of one predictor is not influenced by changes in the reliability of the other predictor (cf. data sets 1, 2, and 3). Furthermore, a predictor's regression coefficient can be corrected for measurement errors by dividing it by the square root of

its reliability. Of course, the regression coefficient can also be corrected for measurement errors in the criterion by dividing it by the square root of criterion reliability.

When the predictors are correlated, the regression coefficient of the less reliable predictor is further reduced in magnitude by increases in the reliability of the other predictor (cf. data sets 4 through 9). Thus, the more reliable predictor receives the greater weight in the fallible model. However, the coefficient of the more reliable predictor may actually become larger in magnitude than its true-score counterpart. For example, compare the coefficients for the data set consisting of Y with a reliability of .30, X_1 with .30, and X_2 with .50 to the data set consisting of Y with a reliability of .30 and the Xs with 1.00.

Consider the special case of one perfectly reliable predictor and one unreliable predictor. If the predictors are uncorrelated, the coefficient of the perfectly reliable predictor is always equal to its true-score counterpart, while the coefficient of the unreliable predictor is always less than its true-score counterpart. If the predictors are correlated, the coefficient of the perfectly reliable predictor is always greater

than its true-score counterpart, while the coefficient of the unreliable predictor is always less. Furthermore, the greater the correlation betweeen the predictors, the greater the magnitude of the discrepancies of the coefficients from their true-score counterparts.

The special case described above is analogous to the problem in the analysis of covariance of an unreliable covariate. If the covariate and experimental treatments are independent (and uncorrelated) as is assumed, then, the regression coefficients (and sum of squares) of the treatments are estimated without bias. Furthermore, although the coefficient of the covariate is negatively biased, it may be corrected to estimate its true-score counterpart by dividing the coefficient with the square root of the covariate's reliability. If the covariate is influenced by the experimental design such that there are correlations between treatment effects and the covariate, however, the regression coefficients of the treatment effects are positively biased and the coefficient of the covariate is negatively biased. In sum, the bias in the effects due to the experimental design and the covariate are in opposite directions. Obviously, this situation is unsatisfactory since totally unrealistic conclusions may be reached in the analysis.

REFERENCES

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Lord, F. M., & Novick, M. R. (1968). Statistical theories of mental test scores. Menlo Park, California: Addison-Wesley.

TABLE 1

Nine Data Sets for the Infallible Model used in the Monte Carlo Study

Data Set	$\rho^2_{\tau \cdot T_1, T_2}$	$\rho^2_{T_1 \tau}$	$\rho^2_{T_2 \tau}$	$\rho^2_{T_1 T_2}$
1	.30	.15	.15	.00
2	.50	.25	.25	.00
3	.80	.40	.40	.00
4	.30	.21	.21	.16
5	.50	.35	.35	.16
6	.80	.56	.56	.16
7	.30	.27	.27	.64
8	.50	.45	.45	.64
9	.80	.72	.72	.64

TABLE 2

The Influence of Measurement Errors on the Squared Multiple Correlation Coefficient

Reliabilities			Infallible Data Sets								
Y	X_1	X_2	1	2	3	4	5	6	7	8	9
.30	.30	.30	.027	.045	.072	.034	.056	.090	.039	.065	.104
.30	.30	.50	.036	.060	.096	.044	.073	.117	.050	.084	.134
.30	.30	.80	.050	.082	.132	.060	.099	.159	.068	.114	.183
.30	.30	1.00	.058	.098	.156	.070	.117	.187	.082	.137	.219
.30	.50	.50	.045	.075	.120	.052	.088	.140	.058	.096	.154
.30	.50	.80	.058	.098	.156	.066	.110	.176	.072	.120	.192
.30	.50	1.00	.068	.112	.180	.075	.126	.201	.083	.139	.222
.30	.80	.80	.072	.120	.192	.076	.127	.204	.079	.132	.211
.30	.80	1.00	.081	.135	.216	.084	.140	.224	.086	.144	.230
.30	1.00	1.00	.090	.150	.240	.090	.150	.240	.090	.150	.240
.50	.30	.30	.045	.075	.120	.056	.094	.150	.065	.109	.174
.50	.30	.50	.060	.100	.160	.073	.122	.195	.084	.139	.223
.50	.30	.80	.082	.138	.220	.099	.165	.264	.114	.190	.304
.50	.30	1.00	.098	.162	.260	.117	.195	.312	.137	.228	.365
.50	.50	.50	.075	.125	.200	.088	.146	.233	.096	.161	.257
.50	.50	.80	.098	.162	.260	.110	.183	.293	.120	.200	.319
.50	.50	1.00	.112	.188	.300	.126	.209	.335	.139	.232	.371
.50	.80	.80	.120	.200	.320	.127	.212	.339	.132	.220	.351
.50	.80	1.00	.135	.225	.360	.140	.233	.372	.144	.240	.384
.50	1.00	1.00	.150	.250	.400	.150	.250	.400	.150	.250	.400

TABLE 2 (continued)

Reliabilities			Infallible Data Sets								
Y	X_1	X_2	1	2	3	4	5	6	7	8	9
.80	.30	.30	.072	.120	.192	.090	.150	.240	.104	.174	.279
.80	.30	.50	.096	.160	.256	.117	.195	.312	.134	.223	.357
.80	.30	.80	.132	.220	.352	.159	.264	.423	.183	.304	.487
.80	.30	1.00	.156	.260	.416	.187	.312	.499	.219	.365	.585
.80	.50	.50	.120	.200	.320	.140	.233	.373	.154	.257	.411
.80	.50	.80	.156	.260	.416	.176	.293	.469	.192	.319	.511
.80	.50	1.00	.180	.300	.480	.201	.335	.536	.222	.371	.593
.80	.80	.80	.192	.320	.512	.204	.339	.543	.211	.351	.562
.80	.80	1.00	.216	.360	.576	.224	.372	.596	.230	.384	.614
.80	1.00	1.00	.240	.400	.640	.240	.400	.640	.240	.400	.640
1.00	.30	.30	.090	.150	.240	.112	.188	.300	.131	.218	.348
1.00	.30	.50	.120	.200	.320	.146	.244	.390	.167	.279	.446
1.00	.30	.80	.165	.275	.440	.198	.330	.529	.228	.381	.609
1.00	.30	1.00	.195	.325	.520	.234	.390	.624	.274	.457	.731
1.00	.50	.50	.150	.250	.400	.175	.292	.467	.193	.321	.514
1.00	.50	.80	.195	.325	.520	.220	.366	.586	.240	.399	.639
1.00	.50	1.00	.225	.375	.600	.251	.418	.670	.278	.463	.741
1.00	.80	.80	.240	.400	.640	.254	.424	.679	.263	.439	.702
1.00	.80	1.00	.270	.450	.720	.279	.466	.745	.288	.480	.767
1.00	1.00	1.00	.300	.500	.800	.300	.500	.800	.300	.500	.800

Note. The infallible data sets are described in Table 1.

TABLE 3

The Influence of Measurement Errors on the Regression Coefficients

Reliabilities			Infallible Data Sets								
Y	X_1	X_2	1	2	3	4	5	6	7	8	9
.30	.30	.30	.116[a]	.150	.190	.123	.158	.200	.126	.162	.205
			.116	.150	.190	.123	.158	.200	.126	.162	.205
.30	.30	.50	.116	.150	.190	.113	.146	.184	.104	.134	.169
			.150	.194	.245	.160	.207	.261	.169	.218	.276
.30	.30	.80	.116	.150	.190	.097	.126	.159	.066	.086	.108
			.190	.245	.310	.205	.265	.336	.229	.295	.373
.30	.30	1.00	.116	.150	.190	.087	.112	.142	.039	.050	.063
			.212	.274	.346	.232	.300	.379	.268	.346	.437
.30	.50	.50	.150	.194	.245	.148	.191	.242	.144	.186	.235
			.150	.194	.245	.148	.191	.242	.144	.186	.235
.30	.50	.80	.150	.194	.245	.129	.166	.211	.097	.126	.159
			.190	.245	.310	.192	.248	.313	.205	.265	.335
.30	.50	1.00	.150	.194	.245	.116	.149	.189	.059	.076	.097
			.212	.274	.346	.218	.282	.356	.251	.324	.410
.30	.80	.80	.190	.245	.310	.170	.220	.278	.155	.200	.254
			.190	.245	.310	.170	.220	.278	.155	.200	.254
.30	.80	1.00	.190	.245	.310	.154	.199	.252	.104	.135	.170
			.212	.274	.346	.196	.253	.320	.210	.271	.343
.30	1.00	1.00	.212	.274	.346	.179	.232	.293	.158	.204	.258
			.212	.274	.346	.179	.232	.293	.158	.204	.258

TABLE 3 (continued)

Reliabilities			Infallible Data Sets								
Y	X_1	X_2	1	2	3	4	5	6	7	8	9
.50	.30	.30	.150	.194	.245	.158	.205	.259	.162	.210	.265
			.150	.194	.245	.158	.205	.259	.162	.210	.265
.50	.30	.50	.150	.194	.245	.146	.188	.238	.134	.172	.218
			.194	.250	.316	.207	.267	.337	.218	.282	.357
.50	.30	.80	.150	.194	.245	.126	.162	.205	.086	.110	.140
			.245	.316	.400	.265	.342	.433	.295	.381	.482
.50	.30	1.00	.150	.194	.245	.112	.144	.183	.050	.064	.081
			.274	.354	.447	.300	.387	.489	.346	.446	.564
.50	.50	.50	.194	.250	.316	.191	.246	.312	.186	.240	.303
			.194	.250	.316	.191	.246	.312	.186	.240	.303
.50	.50	.80	.194	.250	.316	.166	.215	.272	.126	.162	.205
			.245	.316	.400	.248	.320	.404	.265	.342	.433
.50	.50	1.00	.194	.250	.316	.149	.193	.244	.076	.099	.125
			.274	.354	.447	.282	.364	.460	.324	.418	.529
.50	.80	.80	.245	.316	.400	.220	.284	.359	.200	.259	.327
			.245	.316	.400	.220	.284	.359	.200	.259	.327
.50	.80	1.00	.245	.316	.400	.199	.258	.326	.135	.174	.220
			.274	.354	.447	.253	.326	.413	.271	.350	.443
.50	1.00	1.00	.274	.354	.447	.232	.299	.378	.204	.264	.333
			.274	.354	.447	.232	.299	.378	.204	.264	.333

TABLE 3 (continued)

Reliabilities			Infallible Data Sets								
Y	X_1	X_2	1	2	3	4	5	6	7	8	9
.80	.30	.30	.190	.245	.310	.200	.259	.327	.205	.265	.335
			.190	.245	.310	.200	.259	.327	.205	.265	.335
.80	.30	.50	.190	.245	.310	.184	.238	.300	.169	.218	.276
			.245	.316	.400	.261	.337	.427	.276	.357	.451
.80	.30	.80	.190	.245	.310	.159	.205	.259	.108	.140	.177
			.310	.400	.506	.336	.433	.548	.373	.482	.610
.80	.30	1.00	.190	.245	.310	.142	.183	.231	.063	.081	.103
			.346	.447	.566	.379	.489	.619	.437	.564	.714
.80	.50	.50	.245	.316	.400	.242	.312	.394	.235	.303	.383
			.245	.316	.400	.242	.312	.394	.235	.303	.383
.80	.50	.80	.245	.316	.400	.211	.272	.344	.159	.205	.260
			.310	.400	.506	.313	.404	.512	.335	.433	.547
.80	.50	1.00	.245	.316	.400	.189	.244	.309	.097	.125	.158
			.346	.447	.566	.356	.460	.582	.410	.529	.670
.80	.80	.80	.310	.400	.506	.278	.359	.454	.254	.327	.414
			.310	.400	.506	.278	.359	.454	.254	.327	.414
.80	.80	1.00	.310	.400	.506	.252	.326	.412	.170	.220	.278
			.346	.447	.566	.320	.413	.522	.343	.443	.560
.80	1.00	1.00	.346	.447	.566	.293	.378	.478	.258	.333	.422
			.346	.447	.566	.293	.378	.478	.258	.333	.422

TABLE 3 (continued)

Reliabilities			Infallible Data Sets								
Y	X_1	X_2	1	2	3	4	5	6	7	8	9
1.00	.30	.30	.212	.274	.346	.224	.289	.366	.230	.296	.375
			.212	.274	.346	.224	.289	.366	.230	.296	.375
1.00	.30	.50	.212	.274	.346	.206	.266	.336	.189	.244	.308
			.274	.354	.447	.292	.377	.477	.309	.399	.504
1.00	.30	.80	.212	.274	.346	.178	.229	.290	.121	.156	.198
			.346	.447	.566	.375	.484	.612	.417	.539	.682
1.00	.30	1.00	.212	.274	.346	.158	.204	.258	.070	.091	.115
			.387	.500	.632	.424	.547	.692	.489	.631	.798
1.00	.50	.50	.274	.354	.447	.270	.349	.441	.262	.339	.429
			.274	.354	.447	.270	.349	.441	.262	.339	.429
1.00	.50	.80	.274	.354	.447	.235	.304	.384	.178	.230	.290
			.346	.447	.566	.350	.452	.572	.375	.484	.612
1.00	.50	1.00	.274	.354	.447	.211	.273	.345	.108	.140	.176
			.387	.500	.632	.398	.514	.651	.458	.592	.749
1.00	.80	.80	.346	.447	.566	.310	.401	.507	.283	.366	.463
			.346	.447	.566	.310	.401	.507	.283	.366	.463
1.00	.80	1.00	.346	.447	.566	.282	.364	.460	.190	.246	.311
			.387	.500	.632	.357	.461	.584	.383	.495	.626
1.00	1.00	1.00	.387	.500	.632	.327	.423	.534	.289	.373	.471
			.387	.500	.632	.327	.423	.534	.289	.373	.471

Note. The infallible data sets are described in Table 1.

[a] The first entry is the regression coefficient for X_1 and the second is for X_2.